Fluid Dynamics in Astrophysics
Chapter 1: Basic Concept of Fluids

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Fluid Dynamics

- Hydrodynamics (= fluid dynamics) concerns the dynamics of liquid, gases, and plasmas.
- Phenomena are *macroscopic*
- Describe a fluid as a continuous medium with macroscopic quantities (e.g., density, ρ, pressure, p, velocity, …), even though, at a microscopic level, the fluid is composed of particles.

Fluid in the Universe

- Most of the baryonic matter in the Universe can be treated as a fluid
- Hydrodynamics is an *extremely important topic* within astrophysics
- Astrophysical systems can display extremes of density (both low and high) and temperature beyond those accessible in terrestrial laboratories
- Gravity is often a crucial component of the dynamics in astrophysical systems
- The subject of astrophysical hydrodynamics encompasses but significantly extends the study of fluids relevant to terrestrial systems and/or engineers

Fluid in the Universe (cont.)

- In the astrophysical context, the liquid state is not very common (examples: high-pressure environments of planetary surfaces and interiors)
- Our focus will be on the gas phase (or fully ionized gas = plasma)
- Key difference: Gases are more *compressible* than liquids

Fluid in the Universe (cont.)

- *Examples*
  - Interiors of stars, white dwarfs, neutron stars
  - Interstellar medium (ISM), intergalactic medium (IGM), intracluster mediums (ICM)
  - Stellar winds, jets, accretion flows
  - Giant planets
  - Always see shock, turbulence, accretion, outflows

Fluid element

- Use concept of a "*fluid element*"

- This is a region of fluid that

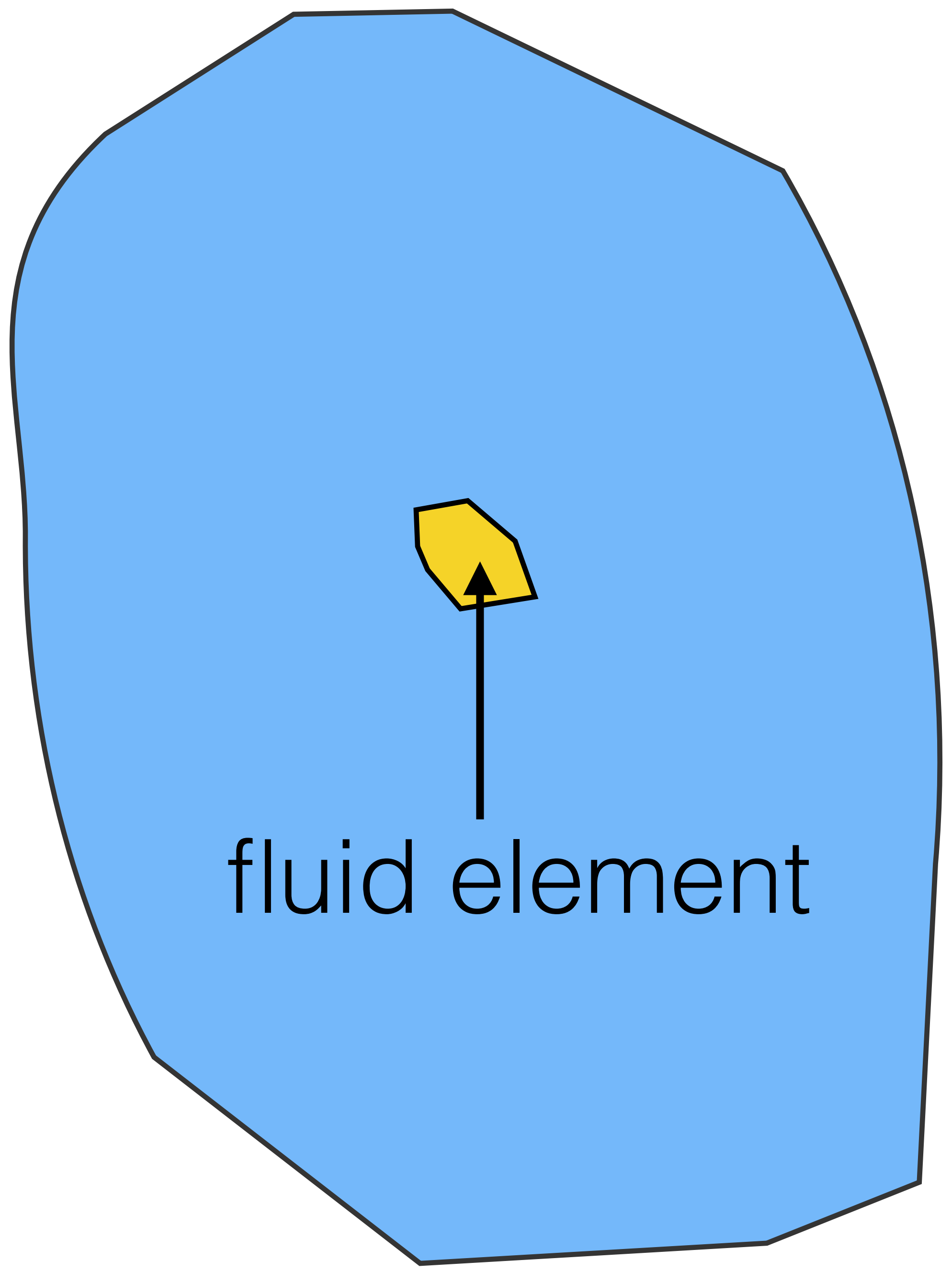
  1. small enough that there are no significant variations of any property q that interests us

$$l_{\rm region} \ll l_{\rm scale} \sim \frac{q}{\nabla q}$$

  2. large enough to contain sufficient particles to be considered in the continuum limit

$$n l_{\rm region}^3 \gg 1$$

  where n is the number density of particles



Collisional and Collisonless Fluids

- In a collisional fluid, any relevant fluid element is large enough such that the constituent particles know about local conditions through interactions with each other,

$$l_{\mathrm{region}} \gg \lambda \quad \textit{(condition for collisional fluid)}$$

- Particles will attain a distribution of velocities that maximizes the entropy of the system at a given temperature

- Collisional fluid at a given density ρ and temperature T will have a distribution of particle velocities and a pressure p

- We can relate ρ, T, and p as the equation of state:

$$p = p(\rho, T)$$

Collisional and Collisonless Fluids (cont.)

- In a collisionless fluid, the particle does *not* interact frequently enough to satisfy the condition:

$$l_{\rm region} \ll \lambda \quad \textit{(condition for collisionless fluid)}$$

- The distribution of particle velocities locally does *not* correspond to the maximum entropy solution, depending on initial conditions and non-local conditions

- Examples of collisionless fluids
  - Stars in a galaxy
  - Grains in Saturn's ring
  - Dark matter
  - ICM (transitional from collisional to collisionless)

Collisional and Collisonless Fluids (cont.)

- Intercluster medium: treat as fully ionised plasma of H(e^-, p).
- Mean free path is set by Coulomb collisions: $$\lambda_e = \frac{3^{3/2}(k_B T_e)^2 \epsilon_0^2}{4\pi^{1/2} n_e e^4 \ln \Lambda}$$

where n_e = electron number density
Λ = ratio of largest to smallest impact parameter

for $T > 4\text{x}10^5$ K we have $\ln\Lambda \sim 40$

if $T_i = T_e$

$$\lambda_e = \lambda_i \approx 23\,\text{kpc} \left(\frac{T_e}{10^8\,\text{K}}\right)^2 \left(\frac{n_e}{10^{-3}\,\text{cm}^{-3}}\right)^{-1}$$

so we have

$$\underbrace{R_{\text{galaxy}} \sim \lambda_e}_{\text{collisionless}} \ll \underbrace{R_{\text{cluster}} \sim}_{\text{collisional}} 1\,\text{Mpc}$$

Validity of the Fluid Approximation

- Generally requires the particle mean free path to be much smaller than scales of interest

- At the microscopic level, this also means that collisions are sufficiently frequent so that particle distribution functions approach Maxwellian (learn in Ch. 3)

	size scale	**particle mean free path**
Air	$\sim 10^{6}$ cm	$\sim 10^{-6}$ cm
Solar interior	$\sim 10^{10}$ cm	$\sim 10^{-8}$ cm (Coulomb)
Solar wind	~AU ~ 10^{13} cm	$\sim 10^{13}$ cm (Coulomb)
Galactic Center	$\sim 10^{13}$cm	$\sim 10^{18}$ cm (Coulomb)
Galaxy cluters	~Mpc ~ 10^{24}cm	$\sim 10^{22}$ cm (Coulomb)

When fluid approximation fails, *kinetic treatment* is generally required (Ch.3)

Astrophysical Fluid Dynamics

- Differs from “laboratory” and/or engineering fluid dynamics in the relative importance of certain effects

- Compressibility
- Gravity
- Magnetic fields

Generally important

- Radiative forces
- Relativistic effects
- Viscosity

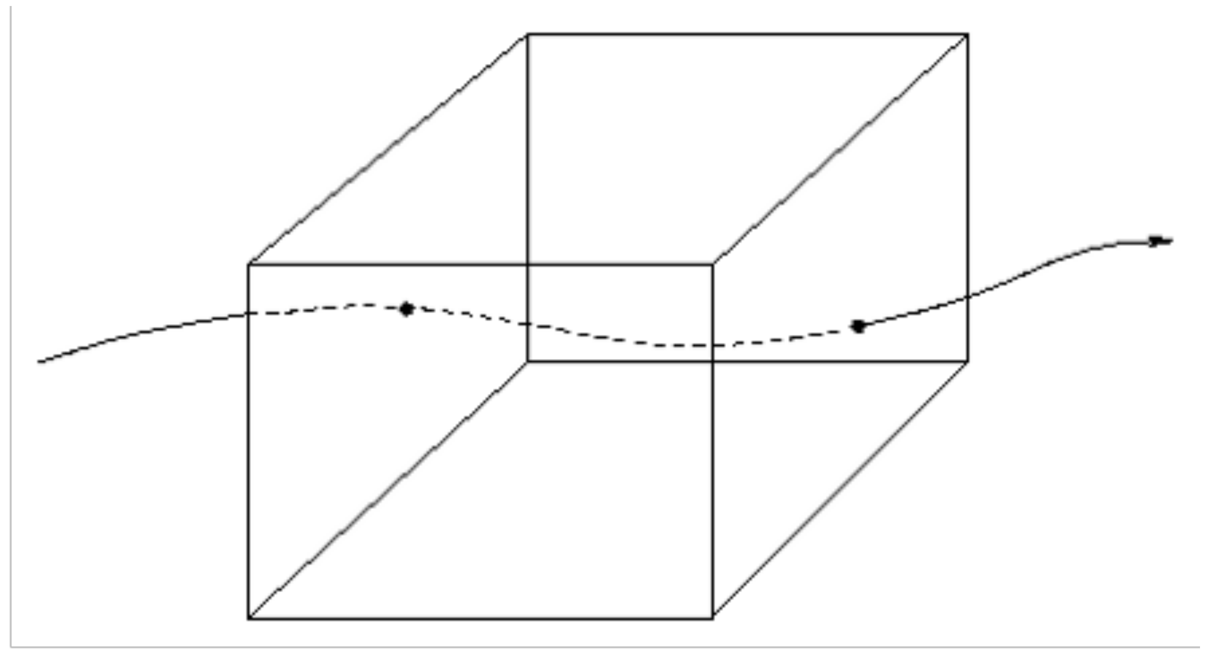
Sometimes important

- Surface tension
- Presence of solid boundaries

Generally unimportant

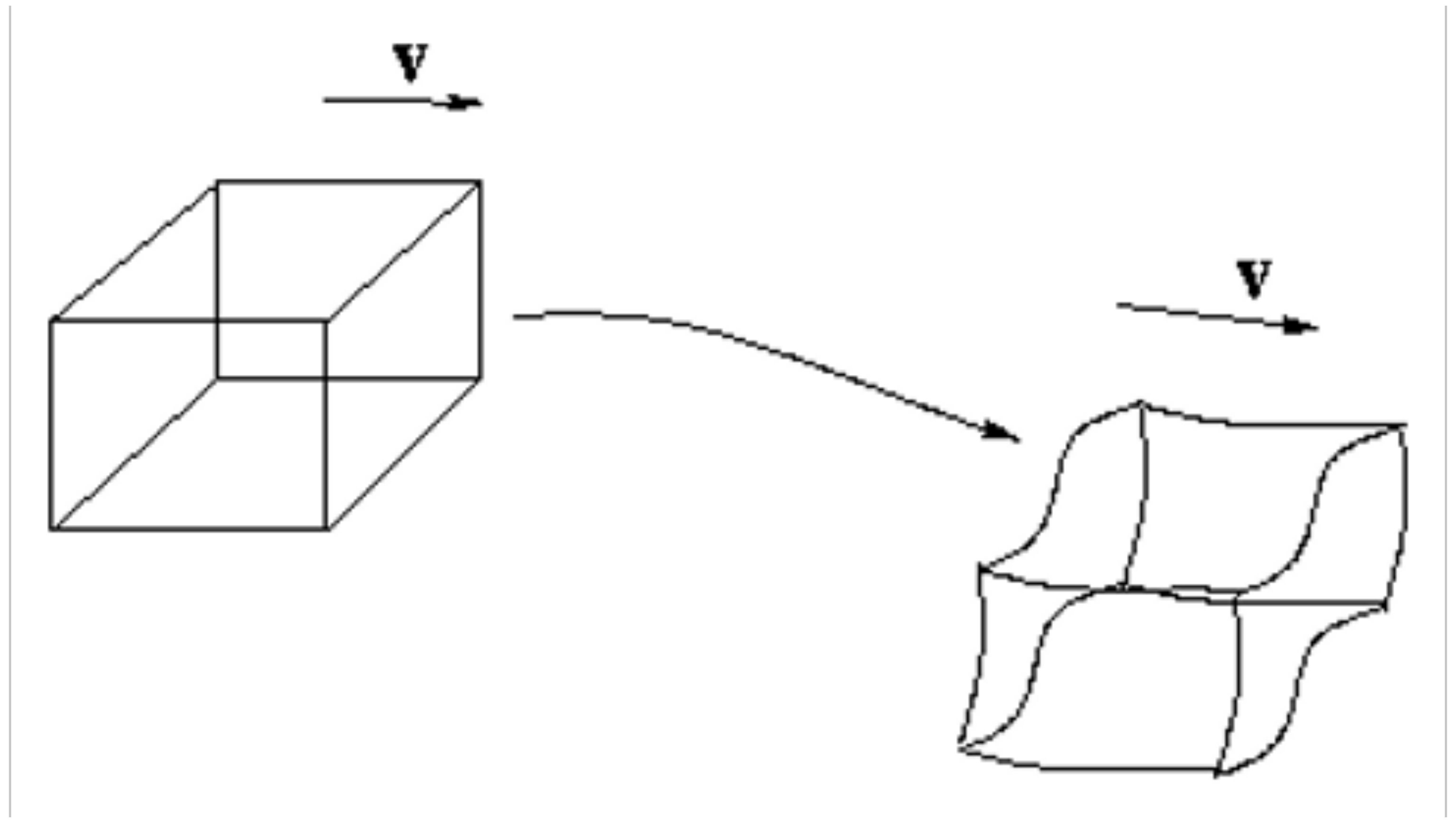
Eulerian vs Lagrangian

- Two main frameworks for understanding fluid flow:



1. Eulerian - one considers the properties of the fluid measured in a frame of reference that is fixed in space. So consider quantities like:

$$\rho(\boldsymbol{r}, t),\ p(\boldsymbol{r}, t),\ T(\boldsymbol{r}, t),\ \boldsymbol{v}(\boldsymbol{r}, t)$$



2. Lagrangian - one considers a particular fluid element and examines the change in the properties of that element. The spatial reference frame is co-moving with the fluid flow

Eulerian vs Lagrangian (cont.)

- Eulerian approach is more useful if the motion of particular fluid elements is not of interest
- Lagrangian approach is useful if we do care about the passage of given fluid elements (e.g., gas parcels that are enriched by metals)
- These two different pictures lead to very different computational approaches to hydrodynamics (grid-based vs Smooth Particle Hydrodynamics)

Eulerian vs Lagrangian (cont.)

- Mathematically, it is straightforward to relate these two pictures.
- Consider a quantity Q in a fluid element at position $\boldsymbol{r}$ and time t
- At time $t+\delta t$ the element will be at the position $r+\delta r$
- Change in Q of the fluid element is

$$\frac{DQ}{Dt} = \lim_{\delta t \to 0} \left[\frac{Q(\boldsymbol{r} + \delta \boldsymbol{r}, t + \delta t) - Q(\boldsymbol{r}, t)}{\delta t} \right]$$

but

$$Q(\boldsymbol{r} + \delta \boldsymbol{r}, t + \delta t) = Q(\boldsymbol{r}, t) + \delta t \cdot \frac{\partial Q}{\partial t} + \delta \boldsymbol{r} \cdot \nabla Q + \mathcal{O}(\delta t^2, |\delta \boldsymbol{r}|^2, \delta t \delta \boldsymbol{r})$$

so

$$\frac{DQ}{Dt} = \lim_{\delta t \to 0} \left[\frac{\partial Q}{\partial t} + \frac{\delta \boldsymbol{r}}{\delta t} \cdot \nabla Q + \mathcal{O}(\delta t, |\delta \boldsymbol{r}|) \right]$$

Eulerian vs Lagrangian (cont.)

which gives us

$$\boxed{\frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + \boldsymbol{v} \cdot \nabla Q}$$

Lagrangian time derivative

(rate of change at moving point)

Eulerian time derivative

(rate of change at fixed point)

“convective” derivative

(change due “transport” of fluid element by bulk motion of fluid itself) = advection

$\boldsymbol{v} \cdot \nabla$: advection operator