Appendix of Astrophysical Hydrodynamics

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A. Conversion of Units

Quantity	units	Symbol	SI	cgs
Length	meter	m	1 m	10^2 cm
Angle	radian	rad		
	degree	$^\circ$	1.7453×10^{-2} rad ($= \pi/180$ rad)	
	arc-minites	$'$	2.9089×10^{-4} rad ($= \pi/10800$ rad)	
	arc-seconds	$''$	4.8481×10^{-6} rad ($= \pi/648000$ rad)	
	milli-arc-seconds	mas	4.8481×10^{-9} rad	
Mass	kilogram	kg	1 kg	10^3 g
Time	second	s		
	minute	min	60 s	
	hour	hr	3.6×10^3 s	
	day	d	8.64×10^4 s	
	year	yr	3.1557×10^7 s	
Frequency	Heltz	Hz	1 s^{-1}	
Force	Newton	N	1 kg m s^{-2}	10^5 dyn
	Dyne	dyn	10^{-5} N	1 g cm s^{-2}
Pressure	Pascal	Pa	1 N m^{-2}	10 dyn cm^{-2}
Energy (Work)	Joule	J	1 N m	10^7 erg
	Erg	erg	10^{-7} J	1 dyn cm
	Electron Volt	eV	1.6022×10^{-19} J	1.6022×10^{12} erg
	Calorie	cal	4.184 J	
Power	Watt	W	1 J s^{-1}	10^7 erg s^{-1}
Electric current	Ampere	A		
Charge	Coulomb	C	1 A s	
Electric Voltage	Volt	V	1 J C^{-1}	
Resistance	Ohm	Ω	1 V A^{-1}	
Magnetic flux	Weber	Wb	1 V s	
Magnetic flux density	Tesla	T	1 Wb m^{-2}	10^4 G
	Gauss	G	10^{-4} T	1 erg cm^{-2} A^{-1}
Temperature	Kelvin	K		

B. Physical Constants

Quantity	Symbol	Value	SI	cgs
Proton mass	m_i	1.6726	10^{-27} kg	10^{-24} g
Electron mass	m_e	9.1095	10^{-31} kg	10^{-28} g
Electron-to-Proton mass ratio	m_i/m_e	1836.2		
Speed of light	c	2.9979	10^{8} m/s	10^{10} cm/s
Gravitational constant	G	6.672	10^{-11} N m^2 kg^{-2}	10^{-8} dyn cm^2 g^{-2}
Boltzmann constant	k_B	1.3807	10^{-23} J K^{-1}	10^{-16} erg K^{-1}
Electronic charge	e	1.6022	10^{-19} C	10^{-20} emu
Permittivity (Dielectric constant)	ϵ_0	8.8542	10^{-12} F m^{-1}	
Permeability of free space	μ_0	4π	10^{-7} H m^{-1}	

Note that $\epsilon_0\mu_0 = 1/c^2$

C. Astronomical Constants

Quantity	Symbol	Value
Astronomical unit	AU	1.4960×10^{11} m
Light year	ly	9.4605×10^{15} m
Parsec	pc	3.0857×10^{16} m
Mass of the Earth	$M_\oplus$	5.972×10^{24} kg
Radius of the Earth	$R_\oplus$	6.3781×10^{6} m
Mass of the Sun	$M_\odot$	1.989×10^{30} kg
Radius of the Sun	$R_\odot$	6.960×10^{8} m

D. Vector Identities

$$\boldsymbol{A}\cdot\boldsymbol{B}\times\boldsymbol{C} = \boldsymbol{B}\cdot\boldsymbol{C}\times\boldsymbol{A} = \boldsymbol{C}\cdot\boldsymbol{A}\times\boldsymbol{B} \tag{D1}$$

$$\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = (\boldsymbol{C}\times\boldsymbol{B})\times\boldsymbol{A} = \boldsymbol{B}(\boldsymbol{A}\cdot\boldsymbol{C}) - \boldsymbol{C}(\boldsymbol{A}\cdot\boldsymbol{B}) \tag{D2}$$

$$\nabla(fg) = f\nabla g + g\nabla f \tag{D3}$$

$$\nabla\cdot(f\boldsymbol{A}) = f\nabla\cdot\boldsymbol{A} + \boldsymbol{A}\cdot\nabla f \tag{D4}$$

$$\nabla \times (f\boldsymbol{A}) = f\nabla \times \boldsymbol{A} + \nabla f \times \boldsymbol{A} \tag{D5}$$

$$\nabla \cdot (\boldsymbol{A} \times \boldsymbol{B}) = \boldsymbol{B} \cdot \nabla \times \boldsymbol{A} - \boldsymbol{A} \cdot \nabla \times \boldsymbol{B} \tag{D6}$$

$$\nabla \times (\boldsymbol{A} \times \boldsymbol{B}) = \boldsymbol{A}(\nabla \cdot \boldsymbol{B}) - \boldsymbol{B}(\nabla \cdot \boldsymbol{A}) + (\boldsymbol{B} \cdot \nabla)\boldsymbol{A} - (\boldsymbol{A} \cdot \nabla)\boldsymbol{B} \tag{D7}$$

$$\boldsymbol{A} \times (\nabla \times \boldsymbol{B}) = (\nabla \boldsymbol{B}) \cdot \boldsymbol{A} - (\boldsymbol{A} \cdot \nabla)\boldsymbol{B} \tag{D8}$$

In particular, slightly re-arranged and specialized

$$(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} = \boldsymbol{B} \cdot \nabla \boldsymbol{B} - \nabla B^2/2 \tag{D9}$$

$$\nabla(\boldsymbol{A} \cdot \boldsymbol{B}) = \boldsymbol{A} \times (\nabla \times \boldsymbol{B}) + \boldsymbol{B} \times (\nabla \times \boldsymbol{A}) + (\boldsymbol{A} \cdot \nabla)\boldsymbol{B} + (\boldsymbol{B} \cdot \nabla)\boldsymbol{A} \tag{D10}$$

$$\nabla \times \nabla f = 0 \tag{D11}$$

$$\nabla \cdot \nabla \times \boldsymbol{A} = 0 \tag{D12}$$

$$\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A} \tag{D13}$$

E. Vector Operators in Various Coordinate Systems

E.1. Cartesian Coordinates

In a Cartesian system (x, y, z)

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) \tag{E1}$$

$$\nabla \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \tag{E2}$$

$$\nabla \times \boldsymbol{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \tag{E3}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \tag{E4}$$

E.2. Cylindrical Coordinates

In a Cylindrical Polar Coordinate system (r, ϕ, z)

$$\nabla f = \left(\frac{\partial f}{\partial r}, \frac{1}{r}\frac{\partial f}{\partial \phi}, \frac{\partial f}{\partial z} \right) \tag{E5}$$

$$\nabla \cdot \boldsymbol{A} = \frac{\partial}{\partial r}(r A_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \tag{E6}$$

$$\nabla \times \boldsymbol{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}, \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}, \frac{1}{r}\frac{\partial}{\partial r}(r A_\phi) - \frac{1}{r}\frac{\partial A_r}{\partial \phi} \right) \tag{E7}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} \tag{E8}$$

E.3. Spherical Polar Coordinates

In a Spherical Polar Coordinate system (r, θ, ϕ)

$$\nabla f = \left(\frac{\partial f}{\partial r}, \frac{1}{r}\frac{\partial f}{\partial \theta}, \frac{1}{r \sin\theta}\frac{\partial f}{\partial \phi} \right) \tag{E9}$$

$$\nabla \cdot \boldsymbol{A} = \frac{\partial}{\partial r^2}(r^2 A_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r \sin\theta}\frac{\partial A_\phi}{\partial \phi} \tag{E10}$$

$$\nabla \times \boldsymbol{A} = \left(\frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\phi) - \frac{1}{r \sin\theta}\frac{\partial A_\theta}{\partial \phi}, \frac{1}{r \sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{1}{r}\frac{\partial}{\partial r}(r A_\phi), \frac{1}{r}\frac{\partial}{\partial r}(r A_\theta) - \frac{1}{r}\frac{\partial A_r}{\partial \theta} \right) \tag{E11}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin\theta}\frac{\partial^2 f}{\partial \phi^2} \tag{E12}$$

$$(\boldsymbol{A}\cdot\nabla)\boldsymbol{B} = \left[A_r\frac{\partial B_r}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_r}{\partial \theta} + \frac{A_\phi}{r\sin\theta}\frac{\partial B_r}{\partial \phi} - \frac{A_\theta B_\theta + A_\phi B_\phi}{r}\right]\hat{r} \tag{E13}$$

$$+ \left[\frac{A_\theta B_r}{r} + A_r\frac{\partial B_\theta}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{A_\phi}{r\sin\theta}\frac{\partial B_\theta}{\partial \phi} - \frac{A_\phi B_\phi}{r}\cot\theta\right]\hat{\theta} \tag{E14}$$

$$+ \left[\frac{A_\phi B_r}{r} + \frac{A_\phi B_\theta}{r}\cot\theta + A_r\frac{\partial B_\phi}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_\phi}{\partial \theta} + \frac{A_\phi}{r\sin\theta}\frac{\partial B_\phi}{\partial \phi}\right]\hat{\phi} \tag{E15}$$